USN | | | | | | | | | | 10MAT11

First Semester B.E. Degree Examination, June/July 2017

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least two from each part.***

PART – A

1 a. Choose the correct answers for the following : **(04 Marks)**

i) The n^{th} derivative of $\cos^2 x$ is

A) $2^n \cos\left(2x + \frac{n\pi}{2}\right)$ B) $2^{n-1} \cos\left(2x + \frac{n\pi}{2}\right)$

C) $2^{n-1} \cos(2x + n\pi)$ D) $2^{n-1} \cos\left(\frac{n\pi}{2}\right)$

ii) The Maclaurin's series of $f(x) = K$ (constant) is

A) $f(x) = K$ B) $f(x) = 0$ C) does not exist D) $f(x) = K!$

iii) The value of C of the Cauchy mean value theorem for $f(x) = e^x$ and $g(x) = e^{-x}$ in [4, 5] is

A) $\frac{5}{2}$ B) $\frac{3}{2}$ C) $\frac{9}{2}$ D) $\frac{1}{2}$

iv) The n^{th} derivative of $y = x^{n-1}.\log x$ is

A) $y_n = \frac{n!}{x}$ B) $y_n = \frac{(n+1)!}{x}$ C) $y_n = \frac{(n-1)!}{x}$ D) $y_n = \frac{n!}{x^2}$

b. If $x = \tan(\log y)$, prove that $(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0$. **(06 Marks)**

c. Expand $\log(\sec x)$ by Maclaurin's series expansion upto the term containing x^4. **(05 Marks)**

d. State and prove Lagrange's mean value theorem. **(05 Marks)**

2 a. Choose the correct answers for the following : **(04 Marks)**

i) $\lim_{x\to\infty} \left[a^{1/x} - 1\right] x$ is of the following form

A) $0 \times \infty$ B) ∞° C) 0^{∞} D) $\infty - \infty$

ii) If S is the arc length of the curve $x = g(y)$ then $\frac{ds}{dy}$ is

A) $\sqrt{1 + y_1}$ B) $\sqrt{1 + y_1^2}$

C) $\sqrt{\left(\frac{dy}{dx}\right)^2 + \left(\frac{dx}{dy}\right)^2}$ D) $\sqrt{1 + \left(\frac{dx}{dy}\right)^2}$

iii) The angle between radius vector and the tangent for the curve $r = a(1 - \cos\theta)$ is

A) $\frac{\theta}{2}$ B) $-\frac{\theta}{2}$ C) $\frac{\pi}{2} + \theta$ D) $\frac{\pi}{2} - \frac{\theta}{2}$

iv) Two polar curves are said to be orthogonal if

A) $\phi_1 \cdot \phi_2 = 0$ B) $\tan\phi_1 \cdot \tan\phi_2 = -1$

C) $\frac{\phi_1}{\phi_2} = \frac{\pi}{2}$ D) $\phi_1 \cdot \phi_2 = -1$

b. If $y = \frac{ax}{a+x}$, then show that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2$ where ρ is the radius of curvature at any point (x, y). **(06 Marks)**

c. Evaluate $\lim_{x\to 0}\left[\frac{\sin x}{x}\right]^{\frac{1}{x^2}}$. **(05 Marks)**

d. Derive an expression for the radius of curvature in polar form. **(05 Marks)**

3 a. Choose the correct answers for the following : **(04 Marks)**

i) If $z = x^2 + y^2$ then $\frac{\partial^2 z}{\partial x \partial y}$ is equal to

A) 0 B) 2 C) 2y D) 2x

ii) The Taylor's series of f(x, y) = xy at (1, 1) is

A) $1 + [(x-1) + (y-1)]$ B) $1 + [(x-1)+(y-1)] + [(x-1)(y-1)]$

C) $(x-1)(y-1)$ D) None of these

iii) If $z = f(x, y)$ then the relative error in z is

A) $\frac{\delta z}{x}$ B) $\delta z - y$ C) $\frac{\delta z}{z}$ D) $z - \delta z$

iv) If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial(r,\theta)}{\partial(x,y)}$ is

A) r B) $\frac{1}{r}$ C) 1 D) −1

b. Find the extreme values of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. **(06 Marks)**

c. If $x = r\cos\theta$, $y = r\sin\theta$, prove that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]$. **(05 Marks)**

d. The diameter and altitude of a can in the form of a right circular cylinder are found to be 4.5 cms and 8.25 cms respectively. The possible error in each measurement is 0.1 cm. Find the approximate error in the volume and lateral surface area. **(05 Marks)**

4 a. Choose the correct answers for the following : **(04 Marks)**

i) The gradient, divergence, curl are respectively

A) scalar, scalar, vector B) vector, scalar vector

C) scalar, vector, vector D) vector, vector, scalar

ii) $\vec{F} = y^2z\hat{i} + z^2x\hat{j} + x^2y\hat{k}$ is

A) constant vector B) solenoidal C) scalar D) none of these

iii) curl grad ϕ is

A) grad curl ϕ B) curl grad ϕ + grad curl ϕ

C) zero D) does not exist

iv) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then curl $\vec{r}$ is

A) 0 B) 1 C) −1 D) ∞

b. If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find $\text{div}\vec{F}$ and $\text{curl}\,\vec{F}$. **(06 Marks)**

c. Prove that $\text{curl}(\phi\vec{F}) = \phi\,\text{curl}\,\vec{F} + \text{grad}\,\phi \times \vec{F}$. **(05 Marks)**

d. Prove that the cylindrical coordinate system is orthogonal. **(05 Marks)**

PART – B

5 a. Choose the correct answers for the following : **(04 Marks)**

i) The value of the integral $\int_0^{\pi/2} \sin^7 x\, dx$ is

A) $\frac{35}{16}$ B) $\frac{16}{35}$ C) $-\frac{16}{35}$ D) $\frac{18}{35}$

ii) $x^2 + y^2 = x^2y^2$ is symmetric about

A) x - axis B) y - axis C) $y = x$ D) All A, B, C

iii) The value of $\int_0^{\pi} \sin^4 x dx$ is

A) $\frac{3\pi}{8}$ B) $\frac{3}{8}$ C) $\frac{\pi}{16}$ D) $\frac{\pi}{4}$

iv) Asymptote to the curve $y^2(a - x) = x^3$ is

A) $y = 0$ B) $x = 0$ C) $x = a$ D) none of these

b. Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx$, $\alpha \geq 0$ using differentiation under integral sign, find $\int_0^1 \frac{x^3 - 1}{\log x} dx$. **(06 Marks)**

c. Obtain reduction formula for $\int_0^{\pi/2} \cos^n x dx$. **(05 Marks)**

d. Find the surface area generated by an arch of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about the x-axis. **(05 Marks)**

6 a. Choose the correct answers for the following : **(04 Marks)**

i) For the differential equation $\left[\frac{d^3y}{dx^3}\right]^2 + \left[\frac{d^2y}{dx^2}\right]^6 + y = x^4$ the order and degree respectively are

A) 2, 6 B) 3, 2 C) 2, 4 D) none of these

ii) The solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

A) $e^x + e^{-y} = c$ B) $e^{-x} + e^{-y} = c$ C) $e^x + e^y = c$ D) $e^{x+y} = c$

iii) The integrating factor of the differential equation $\frac{dx}{dy} + Px = Q$ where P, Q are functions Y is

A) $e^{\int pdy}$ B) $e^{\int pdx}$ C) $e^{\int Qdy}$ D) none of these

iv) If the differential equation of the given family remains unaltered after replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$ then given family of curves is said to be

A) not orthogonal B) self orthogonal C) reciprocal D) none of these

b. Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(06 Marks)**

c. Solve $\left[x\tan\left(\frac{y}{x}\right) - y\sec^2\left(\frac{y}{x}\right)\right]dx + x\sec^2\left(\frac{y}{x}\right)dy = 0$. **(05 Marks)**

d. Find the orthogonal trajectory of $r^n = a^n \sin n\theta$. **(05 Marks)**

7 a. Choose the correct answers for the following : (04 Marks)

i) Which of the following is not an elementary transformation

A) adding two columns B) adding two rows

C) squaring all elements of the matrix D) multiplying a row by a non-zero number

ii) The exact solution of the system of equations $10x+y+z=12$, $x+10y+z=12$, $x+y+10z=12$ by inspection is

A) (-1, 1, 1) B) (-1, -1, -1) C) (1, 1, 1) D) (0, 0, 0)

iii) If r is the rank of the matrix [A] of order $m \times n$ then r is

A) $r \leq$ minimum of (m, n) B) $r \leq n$

C) $r > n$ D) $r \geq m$

iv) Which of the following is in the normal form

A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ D) all of these

b. Find the rank of the following matrix by reducing it to the normal form

$$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$ (06 Marks)

c. Find the value of K such that the following system equations possess a non-trivial solution $4x+9y+z=0$, $Kx+3y+Kz=0$, $x+4y+2z=0$. (05 Marks)

d. Solve the following system of equations by Gauss Jordan method:
$x+y+z=9$, $x-2y+3z=8$, $2x+y-z=3$ (05 Marks)

8 a. Choose the correct answers for the following : (04 Marks)

i) A square matrix A is called orthogonal if

A) $A=A^2$ B) $A^1=A$ C) $AA^1=I$ D) none of these

ii) The eigen values of the matrix A exist if

A) A is a square matrix B) A is singular matrix

C) A is any matrix D) A is null matrix

iii) The matrix of the quadratic form $a_{11}x_1^2+2a_{12}x_1x_2+a_{22}x_2^2$ is

A) $\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{11} \end{bmatrix}$ B) $\begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}$ C) $\begin{bmatrix} 1 & a_{11} \\ a_{11} & 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

iv) If the eigen vector is (1, 1, 1) then its normalized form is

A) $\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$ B) $\left(\frac{1}{\sqrt{2}},0,-\frac{1}{\sqrt{2}}\right)$ C) $\left(-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ D) $\left(-\frac{1}{\sqrt{3}},-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

b. Find the eigen values and eigen vector corresponding to the largest eigen value of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$ (06 Marks)

c. Diagonalize the matrix, $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (05 Marks)

d. Reduce the quadratic form $x_1^2+2x_2^2-7x_3^2-4x_1x_2+8x_2x_3$ into sum of squares. (05 Marks)